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Monetary Policy and Inequality under Endogenous Financial Segmentation

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Abstract

The empirical evidence on how monetary policy affects inequality is mixed. We propose a model with endogenously segmented financial markets which can reconcile this contrasting empirical evidence. In addition to the conventional income composition channel (intensive margin), monetary policy in our model also affects inequality through a *financial-inclusion margin* channel (extensive margin) by altering the extent of financial participation. We show analytically that the two channels impact inequality in opposite ways and that the net impact depends upon (a) the extent of financial market exclusion and (b) the density of households at the margin of financial market participation in an economy.

KEYWORDS: Inequality, Monetary Policy, Segmented Financial Markets JEL CLASSIFICATION: D63, E52, G10

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1 Introduction

The recent empirical evidence on how monetary policy affects inequality is mixed. While one strand of literature finds that contractionary monetary policy increases inequality (for instance, Coibion et al. (2017), Guerello (2018), Furceri et al. (2018)), another strand of empirical evidence suggests that it decreases income inequality (for instance, Cloyne et al. (2016), Inui et al. (2017), O'Farrell et al. (2016)).

We attempt to show that there is a way to reconcile these two contrasting empirical evidence in a model with endogenous financial market participation. Contrary to the existing theoretical literature on financial market and inequality (for instance, Favilukis (2013), Areosa & Areosa (2016)) we endogenize the financial market participation and build an economy with equity markets where monetary policy can impact inequality in opposite ways.

2 The Model Economy

2.1 Environment

We introduce heterogeneity in equity market participation as in Alvarez et.al (2009). Shocks to money growth μ_t , and dividend growth rate g_t are the two sources of uncertainty in the model; $s_t \equiv (\mu_t, g_t)$ denotes the aggregate event in period t with s_0 as given and $s^t = (s_1, s_2, s_3, \dots, s_t)$ denotes the history of states which have occurred until and including period t.

2.1.1 Households

Every household has a brokerage account and a checking account. The brokerage account is used for asset market transactions while the checking account is used for goods market transactions. Money can be transferred across the two accounts. However, every household faces an idiosyncratic fixed cost fixed cost γ_i , to transfer money from the brokerage account to the checking account, where the distribution of these fixed costs is governed by a invariant density function $f(\gamma)$. To keep our mechanism sharp, our model takes an extreme view where the households are otherwise identical but only differ in the fixed costs they face. Since the households only differ in these fixed costs, we use γ to index a household.

Brokerage Account: The asset markets open at time t = 0, and the household learns about its fixed cost. The households have initial asset holdings, $\bar{B}(\gamma)$ in their brokerage account. With these

asset holdings, the households trade shares, $S_0(\gamma)$ and a complete set of one-period state contingent claims, $B(s^1, \gamma)$ issued by the financial intermediary. The accounting constraint for the household's brokerage account at time t = 0 is, $\bar{B}(\gamma) = j(s_0)S(s_0, \gamma) + \int_{s_1} q(s^1)B(s^1, \gamma)ds_1$ and for time $t \ge 1$ is

$$B(s^{t},\gamma) + P(s^{t})\{j(s^{t}) + \epsilon_{t}\}S(s^{t-1},\gamma) = \int_{s_{t+1}} q(s^{t},s_{t+1})B(s^{t},s_{t+1},\gamma)ds_{t+1} + P(s^{t})j(s^{t})S(s^{t},\gamma) + P(s^{t})[x(s^{t},\gamma) + \gamma]z(s^{t},\gamma)$$
(1)

where $P(s^t)$ denotes the aggregate price level, $B(s^t, s_{t+1}, \gamma)$ denotes the amount of bonds purchased by a household at time t at a price $q(s^t, s_{t+1})$, which will pay off a dollar in time t + 1 if state s_{t+1} occurs; $S(s^{t-1}, \gamma)$ denote the stocks purchased in period t - 1; $j(s^t)$ and ϵ_t respectively denote the real price and real dividend respectively of a stock realized at the beginning of period t. If the household makes a transfer of $P(s^t)x(s^t, \gamma)$ from the brokerage account to the checking account in period t, then it has to pay a fixed cost of γ in real terms; $z(s^t, \gamma)$ is an indicator function which takes a value of 1, if the household makes such a transfer, or 0 otherwise. Households with $z(s^t, \gamma) = 1$, at period t, are referred to as financially included (FI) households, and those with $z(s^t, \gamma) = 0$, are referred to as financially excluded (FE).

Checking Account: At $t \ge 1$, the goods market opens and the household can choose to purchase consumption goods $c(s^t, \gamma)$ through the cash in the checking account. In period $t \ge 1$, the household enters with cash $M(s^{t-1}, \gamma)$ in the checking account, carried over from the previous period. If it also makes a transfer from the brokerage account in period $t \ge 1$, then the additional cash balances are $P(s^t)x(s^t, \gamma)$. The flow constraint for the checking account is

$$P(s^t)c(s^t,\gamma) = M(s^{t-1},\gamma) + P(s^t)x(s^t,\gamma)z(s^t,\gamma)$$

$$\tag{2}$$

Each household also receives an endowment \tilde{y} which is invariant with time and homogenous across the household population. The household sells this endowment in the goods market at the end of the period t, and this become the nominal cash balances that it carries over to the next period t + 1. Therefore,

$$M(s^t, \gamma) = P(s^t)\tilde{y} \tag{3}$$

If $h(s^t)$ is the probability distribution over history s^t , then a household's problem is now to choose

 $\{c(s^t,\gamma), x(s^t,\gamma), z(s^t,\gamma), B(s^t,s_{t+1},\gamma), S(s^t,\gamma)\}_{t=1}^\infty$ to maximize

$$\max\sum_{t=1}^{\infty} \int_{s^t} \beta^t U(c(s^t, \gamma))h(s^t) ds^t$$
(4)

subject to the constraints of equation (1), (2) and (3).

2.1.2 Monetary Policy

The government conducts open market operations to exchange money for one-period state-contingent bonds $B(s^t)$. At time t = 0, the budget constraint of the government is given by $B = \int_{s_1} q(s_1)B(s^1)ds_1$. For $t \ge 1$, the budget constraint is

$$B(s^{t}) + M_{t-1} = M_t + \int_{s_{t+1}} q(s^{t}, s_{t+1}) B(s^{t}, s_{t+1}) ds_{t+1}$$
(5)

with $M_0 > 0$ as given. Monetary policy is specified through the money growth rate in gross terms as $\mu_t = M_t/M_{t-1}$.

2.1.3 Financial Intermediary

The financial intermediary bundles the government bonds into state-contingent bonds while making zero profits such that for all $t \ge 0$

$$\int_{s_{t+1}} \int_{\gamma} q(s^t, s_{t+1}) B(s^t, s_{t+1}, \gamma) f(\gamma) d\gamma ds_{t+1} = \int_{s_{t+1}} q(s^t, s_{t+1}) B(s^t, s_{t+1}) ds_{t+1}$$
(6)

2.2 Equilibrium

The overall resource constraint for the economy is (proved in Appendix A.1)

$$\int_{\gamma} \left[c(s^t, \gamma) + \gamma z(s^t, \gamma) \right] f(\gamma) d\gamma = Y_t \tag{7}$$

where

$$Y_t = \int_{\gamma} \tilde{y} f(\gamma) d\gamma + \int_{\gamma} \epsilon_t S(s^{t-1}, \gamma) f(\gamma) d\gamma$$
(8)

The total equity holdings is normalized to a measure of 1, therefore, for the equity market to clear

$$\int_{\gamma} S(s^{t-1}, \gamma) f(\gamma) d\gamma = 1 \tag{9}$$

The bonds market clear through equation (6). The money market clears when:

$$\int_{\gamma} \{ M(s^{t-1}, \gamma) + P(s^t) \left[x(s^t, \gamma) + \gamma \right] z(s^t, \gamma) \} f(\gamma) d\gamma = M_t$$
(10)

It is easy to see the quantity equation for money holds such that $Y_t = \frac{M_t}{P(s^t)}$. Holding Y_t as fixed, the (gross) inflation rate, $\pi_t (\equiv \frac{P(s^{t+1})}{P(s^t)})$ is simply equal to (gross) rate of money growth $\mu_t (\equiv \frac{M_{t+1}}{M_t})$.

2.3 Segmentation Dynamics

Lemma 1. At any time t, the financially included households identically consume c_t^{FI} and the financially excluded households identically consume c_t^{FE} .

Proof. Refer to Appendix(A.2)

Lemma 2. All households which face a cost, $\gamma_i \leq \hat{\gamma}$ will be financially included and the households for which $\gamma_i > \hat{\gamma}$ will be financially excluded at time t, where $\hat{\gamma}$ is given by

$$\frac{\left[U(c_t^{FI}-U(c_t^{FE})\right]-U'(c_t^{FI})[c_t^{FI}-c_t^{FE}]}{U'(c_t^{FI})}$$

Proof. Refer to Appendix(A.3)

Consequently, the proportion of financially included and excluded households in the economy is respectively given by $\int_{0}^{\widehat{\gamma}_{t}} f(\gamma) d\gamma$ and $1 - \int_{0}^{\widehat{\gamma}_{t}} f(\gamma) d\gamma$.

Lemma 3. Expansionary monetary policy increases the proportion of financially included households in the economy

$$\frac{d}{d\mu_t} [\int_0^{\widehat{\gamma_t}} f(\gamma) d\gamma] > 0$$

Proof. Refer to Appendix (A.4) for detailed proof.

Intuitively, with an increase in the money supply, the inflation tax on the FE households increases, consequently, the incentive on the FE household at the margin is to convert their status to FI and as a result, the number of FI households increase.

3 Results & Discussion

3.1 Endogenous Inequality

The literature on inequality has, besides income, also focussed on heterogeneity in consumption as a measure of inequality (Krueger and Perri (2006)). Following this literature, we construct Figure 1 where the x-axis of the $\triangle ABC$ is the proportion of FI and FE households, and the y-axis is simply the respective aggregate consumption share relative to the total endowment available for consumption in the economy.



Figure 1: Endogenous Inequality and Monetary Policy

We define the consumption gini as the ratio of areas of $\triangle AOC / \triangle ABC$ in Figure 1:

$$gini_{t} = area(\triangle AOC)/area(\triangle ABC)$$
$$= \left[1 - \int_{0}^{\widehat{\gamma}_{t}} f(\gamma)d\gamma\right] - \left[1 - \int_{0}^{\widehat{\gamma}_{t}} f(\gamma)d\gamma\right] * \left[\frac{1}{\mu_{t}}\right]$$
(11)

To understand how inequality is impacted with monetary policy, differentiate (11) and segregate the terms to get:

$$\frac{\partial gini_t}{\partial \mu_t} = \left[\underbrace{\left[1 - \int_0^{\widehat{\gamma_t}} f(\gamma) d\gamma\right] \frac{1}{\mu_t^2}}_{\text{(a) Intensive margin}}\right] - \left[\underbrace{\left(1 - \frac{1}{\mu_t}\right) f(\widehat{\gamma_t}) \frac{\partial \widehat{\gamma_t}}{\partial \mu_t}}_{\text{(b) Extensive margin}}\right]$$
(12)

The RHS of (12) has two effects:

- (a) Intensive margin effect: $[1 \int_0^{\widehat{\gamma}_t} f(\gamma) d\gamma] \frac{1}{\mu_t^2}$. The term in brackets is the proportion of financially excluded households who are simply consuming their real balances. Expansionary monetary policy increases inflation tax, thereby reducing the consumption share of non-participants and hence increasing inequality. In Figure 1 we denote this as the area of $\Diamond AOCO'$, which gets added to the initial area of $\triangle AOC$.
- (b) Extensive margin effect: $(1 \frac{1}{\mu_t})f(\widehat{\gamma_t})\frac{d\widehat{\gamma_t}}{d\mu_t}$. The term $f(\widehat{\gamma_t})\frac{d\widehat{\gamma_t}}{d\mu_t}$ is the proportion of households at the margin of inclusion, $(1 - \frac{1}{\mu_t})$ is the increment in the consumption share of such marginal households when they become financially included, and this increment reduces inequality. Figure 1 we denote this as the area of $\Diamond AOCO''$, which gets subtracted from the initial area of $\triangle AOC$.

Proposition 1. Monetary policy has two contrasting impacts on inequality: intensive effect and extensive effect. An expansionary monetary policy, for instance, increases inequality through intensive margin effect but decreases inequality through extensive margin effect.

Proof. The first term in the RHS of (12) is the proportion of FE households, which being a nonnegative number, increases inequality. The second term, is also positive because From Lemma 3 we know that $\frac{d\hat{\gamma}_t}{d\mu_t} > 0$. But since the second term enters the RHS of (12) with a negative sign, it decreases inequality.

Proposition 2. The net effect of monetary policy on inequality depends upon the extent of financial exclusion and the density of households in the neighborhood of the margin of financial inclusion.

Proof. Using (12), it is easy to see that $\frac{\partial gini_t}{\partial \mu_t} < (or, >)0$ when

$$\frac{\partial \widehat{\gamma_t}}{\partial \mu_t} > (or, <) \underbrace{\frac{\overbrace{[1 - \int_0^{\widehat{\gamma_t}} f(\gamma) d\gamma]}}{\underbrace{f(\widehat{\gamma_t})}}_{density at exclusion margin}} \frac{1}{(\mu_t)(\mu_t - 1)}$$

which proves Proposition 2.

3.2 Counterfactual: Exogenous Participation

Suppose that our participation margin channel was absent (as in Areosa and Areosa (2016)). Then inequality would simply have been

$$gini_t^{exogenous} = (1 - \lambda) - (1 - \lambda)(1/\mu_t)$$

Consequently,

$$\frac{\partial gini_t^{exogenous}}{\partial \mu_t} = (1 - \lambda)(1/\mu_t^2) > 0$$

Monetary policy in the case of exogenous participation has a uni-directional impact on inequality. For instance, expansionary monetary policy would always increase inequality through the traditional income composition channel where only the financial market participants who hold financial assets will earn capital income on monetary injection. In our case, however, expansionary monetary policy also increases the proportion of financial market participants, thereby allowing some of the previously financially excluded households to augment their income through capital gains, and this additional channel brings down inequality.

4 Conclusion

We extend the extant theoretical literature on monetary policy and inequality by proposing a financial-inclusion margin channel in a setting where financial market participation is endogenous. We show that, in line with the empirical evidence, monetary policy can have ambiguous impact on inequality. An expansionary monetary policy, for instance, raises the inflation tax levied on each financially excluded household's real balances. In equilibrium, asset prices adjust to redistribute the inflation tax revenues to financially included households, thereby increasing inequality. However, as inflation varies, so does the benefit of financial market participation, and that changes the fraction of financially included households. At the margin, an expansionary monetary policy by incentivizing participation due to increased consumption, increases the participation fraction, and therefore, reduces inequality. This is the financial-inclusion margin channel.

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A Appendix

A.1 Market Equilibrium

Aggregate equation (1) over γ , substitute equation (6) and rearrange to:

$$\begin{split} B(s^{t}) &- \int_{s_{t+1}} q(s^{t}, s_{t+1}) B(s^{t}, s_{t+1}) ds_{t+1} = \int_{\gamma} [P(s^{t})j(s^{t})S(s^{t}, \gamma) + P(s^{t})[x(s^{t}, \gamma) + \gamma]z(s^{t}, \gamma) \\ &- P(s^{t})\{j(s^{t}) + \epsilon_{t}\}S(s^{t-1}, \gamma)]f(\gamma)d\gamma \end{split}$$

Substitute from equation (5) & (9) to get

$$M_t - M_{t+1} = \int_{\gamma} P(s^t) [x(s^t, \gamma) + \gamma] z(s^t, \gamma) f(\gamma) d\gamma - P(s^t) \epsilon_t$$
(13)

Add $\gamma z(s^t, \gamma)$ term on either side of equation (2) and aggregate over the households to get

$$\int_{\gamma} P(s^{t})[c(s^{t},\gamma)+\gamma]z(s^{t},\gamma)f(\gamma)d\gamma = \int_{\gamma} [M(s^{t-1},\gamma)+P(s^{t})(x(s^{t},\gamma)+\gamma)z(s^{t},\gamma)]f(\gamma)d\gamma \quad (14)$$

Substitute in (14), equations (13), (3) and (8), to get (7).

A.2 Proof of Lemma 1

Construct a Lagrangian:

$$\begin{split} \breve{\mathbf{G}} &= \sum_{t=1}^{\infty} \int_{s^{t}} \beta^{t} U(c(s^{t},\gamma)) h(s^{t}) ds^{t} \\ &+ \Lambda(\gamma) \{ \bar{B}(\gamma) - \sum_{t=1}^{\infty} \int_{s^{t}} Q(s^{t}) \{ P(s^{t}) [x(s^{t},\gamma) + \gamma] z(s^{t},\gamma) + P(s^{t}) [j(s^{t}) S(s^{t},\gamma) \\ &- j(s^{t}) S(s^{t-1},\gamma) - \epsilon_{t} S(s^{t-1},\gamma)] ds^{t} - j(s_{0}) S(s_{0},\gamma) \} \\ &+ \sum_{t=1}^{\infty} \int_{s^{t}} \Theta(s^{t},\gamma) \{ \frac{M(s^{t-1},\gamma)}{P(s^{t})} + x(s^{t},\gamma) z(s^{t},\gamma) - c(s^{t},\gamma) \} ds^{t} \\ &+ \sum_{t=1}^{\infty} \int_{s^{t}} \vartheta(s^{t},\gamma) \{ P(s^{t}) \tilde{y} - \frac{M(s^{t},\gamma)}{P(s^{t})} \} ds^{t} \end{split}$$
(15)

where $\Lambda(\gamma), \Theta(s^t, \gamma)$ and $\vartheta(s^t, \gamma)$ denote the Lagrangian multiplier for (1), (2) and (3) respectively and $Q(s^t) = \prod_{j=1}^t q(s^t)$. Standard transversality condition $\lim_{t \to \infty} \int_{s^t} Q(s^t) B(s^t, \gamma) ds^t = 0$

0; $\lim_{t \to \infty} \int_{s^t} Q(s^t) S(s^t, \gamma) ds^t = 0$

Since the households are ex-ante identical, the Lagrangian multiplier $\Lambda(\gamma) = \Lambda$. FOCs:

$$\begin{aligned} c(s^t,\gamma) &: \beta^t U'\left[c(s^t,\gamma)\right] h(s^t) = \Theta(s^t,\gamma) \\ x(s^t,\gamma) &: \Theta(s^t,\gamma) z(s^t,\gamma) = \Lambda Q(s^t) P(s^t) \\ \Theta(s^t,\gamma) &: c(s^t,\gamma) = \frac{M(s^{t-1},\gamma)}{P(s^t)} + x(s^t,\gamma) z(s^t,\gamma) \end{aligned}$$

For FI households $z(s^t, \gamma) = 1$, $\beta^t U' [c_t(s^t, \gamma)] h(s^t) = \Lambda Q(s^t) P(s^t)$. The RHS does not depend upon idiosyncratic γ , implying the FI households identically consume $c_t^{FI}(s^t)$. For FE households $z(s^t, \gamma) = 0$, $c(s^t, \gamma) = \frac{M(s^{t-1}, \gamma)}{P(s^t)}$. Since $M(s^{t-1}, \gamma) = P(s^{t-1})\tilde{y}$, $c_t^{FE}(s^t) = P(s^{t-1})\tilde{y}/P(s^t)$.

A.3 Proof of Lemma 2

Use the above results in (15) and write the associated kernel as:

$$\beta^{t} \{ U(c_{t}^{FI})z(s^{t},\gamma) + U(c_{t}^{FE})(1-z(s^{t},\gamma)) \} h(s^{t})ds^{t} - \Lambda Q(s^{t}) \{ P(s^{t})\{c_{t}^{FI} - c_{t}^{FE} + \gamma \} z(s^{t},\gamma)ds^{t} \} h(s^{t})ds^{t} - \Lambda Q(s^{t})\{ P(s^{t})\{c_{t}^{FI} - c_{t}^{FE} + \gamma \} z(s^{t},\gamma)ds^{t} \} h(s^{t})ds^{t} - \Lambda Q(s^{t})\{ P(s^{t})\{c_{t}^{FI} - c_{t}^{FE} + \gamma \} z(s^{t},\gamma)ds^{t} \} h(s^{t})ds^{t} + \Lambda Q(s^{t})\{ P(s^{t})\{c_{t}^{FI} - c_{t}^{FE} + \gamma \} z(s^{t},\gamma)ds^{t} \} h(s^{t})ds^{t} + \Lambda Q(s^{t})\{ P(s^{t})\{c_{t}^{FI} - c_{t}^{FE} + \gamma \} z(s^{t},\gamma)ds^{t} \} h(s^{t})ds^{t} + \Lambda Q(s^{t})\{ P(s^{t})\{c_{t}^{FI} - c_{t}^{FE} + \gamma \} z(s^{t},\gamma)ds^{t} \} h(s^{t})ds^{t} + \Lambda Q(s^{t})\{ P(s^{t})\{c_{t}^{FI} - c_{t}^{FE} + \gamma \} z(s^{t},\gamma)ds^{t} \} h(s^{t})ds^{t} + \Lambda Q(s^{t})\{ P(s^{t})\{c_{t}^{FI} - c_{t}^{FE} + \gamma \} z(s^{t},\gamma)ds^{t} \} h(s^{t})ds^{t} + \Lambda Q(s^{t})\{ P(s^{t})\{c_{t}^{FI} - c_{t}^{FE} + \gamma \} z(s^{t},\gamma)ds^{t} \} h(s^{t})ds^{t} + \Lambda Q(s^{t})\{ P(s^{t})\{c_{t}^{FI} - c_{t}^{FE} + \gamma \} z(s^{t},\gamma)ds^{t} \} h(s^{t})ds^{t} + \Lambda Q(s^{t})\{ P(s^{t})\{c_{t}^{FI} - c_{t}^{FE} + \gamma \} z(s^{t},\gamma)ds^{t} \} h(s^{t})ds^{t} + \Lambda Q(s^{t})\{ P(s^{t})\{c_{t}^{FI} - c_{t}^{FE} + \gamma \} z(s^{t},\gamma)ds^{t} \} h(s^{t})ds^{t} + \Lambda Q(s^{t})\{ P(s^{t})\{c_{t}^{FI} - c_{t}^{FE} + \gamma \} z(s^{t})ds^{t} \} h(s^{t})ds^{t} + \Lambda Q(s^{t})\{c_{t}^{FI} - c_{t}^{FE} + \gamma \} h(s^{t})ds^{t} \} h(s^{t})ds^{t} + \Lambda Q(s^{t})ds^{t} + \Lambda Q(s^{t})ds^{t}$$

The increment to the Lagrangian, for $z(s^t, \gamma) = 1$ and 0 is respectively given by $\beta^t \{U(c_t^{FI})h(s^t)ds^t - \Lambda Q(s^t)P(s^t)(c_t^{FI} - c_t^{FE} + \gamma)\}ds^t$ and $\beta^t U(c_t^{FE})h(s^t)ds^t$. Subtracting the latter from former, and using $U'(c_t^{FI})h(s^t)ds^t = \Lambda Q(s^t)P(s^t)ds^t$ we get that a household will be financially included only when

$$\left[U(c_t^{FI}) - U(c_t^{FE})\right] - U'(c_t^{FI})(c_t^{FI} + \gamma_i - c_t^{FE}) \ge 0$$

which proves Lemma 2.

A.4 Proof of Lemma 3

To prove Lemma 2 we need $\frac{\partial \hat{\gamma}_t}{\partial \mu_t} > 0$. Differentiating (7) and substituting the result of Lemma 1, we get

$$\frac{\partial \widehat{\gamma_t}}{\partial \mu_t} = \frac{\{U'(\tilde{y}/\mu_t) - U'(c_t^{FI})\}\tilde{y}/\mu_t^2 - U''(c_t^{FI})\{c_t^{FI} + \widehat{\gamma_t} - \tilde{y}/\mu_t\}(\tilde{y}/\mu_t^2)\frac{(1 - F(\widehat{\gamma_t}))}{F(\widehat{\gamma_t})}}{U'(c_t^{FI}) - U''(c_t^{FI})\{c_t^{FI} + \widehat{\gamma_t} - \tilde{y}/\mu_t\}^2 f(\widehat{\gamma_t})/F(\widehat{\gamma_t})}$$
(16)

Since $c^{FI} > \tilde{y}/\mu$ for $\mu > 1$, for a concave utility, $U'(\tilde{y}/\mu_t) > U'(c_t^{FI})$ and $U''(c^{FI}) < 0$. Together, this proves Lemma 3.